

equation is not independent of the others, but if we make $x = \sin \delta$, $y = \cos \delta \cos \alpha$, $z = \cos \delta \sin \alpha$, we shall obtain three linear equations in x, y, z which can be solved, and thus α and δ are found without any ambiguity whatever.

All the ordinary formulæ used in connection with the different instruments named can be deduced as particular cases of the general equation (i).

In general, there are no real values of R and R' when the instrument is directed to the pole of circle A . In such a case R would have to be set on one of the imaginary circular points at infinity.

2nd January 1908.

The Perturbations of Halley's Comet in the Past. Second Paper.
The Apparition of 1222. By P. H. Cowell, M.A., F.R.S.,
 and A. C. D. Crommelin, B.A.

In the first paper of this series we identified the comet of October 1301 with Halley's, and found the value $44''.858$ for the mean daily motion at that epoch. We have now completed (with the aid of Mr. F. R. Cripps) the calculation of the perturbations by Jupiter and Saturn for the preceding revolution. As a first approximation, Hind's date (mid-July 1223) was assumed for the preceding perihelion passage, and on this assumption the results were as follows:—

Planet.	Limit of u .	dn .	$d\omega$.	$d\zeta$.
Jupiter	$0^\circ-90^\circ$	$-'.2261$	$-357''$	$-6550''$
„	$90-270$	$-.0823$	-35	$+6001$
„	$270-360$	$+.5624$	-183	-353
Saturn	$0-90$	$-.0619$	$+12$	-1784
„	$90-270$	$+.0636$	-39	$+1602$
„	$270-360$	$-.2411$	$+15$	$+41$
Sum	...	$+.0146$	-587	-1043

Hence mean motion in 1223 = $44''.858 - 0''.015 = 44''.843$

and calculated period = $\frac{1296000'' + 1043''}{44''.843} = 28924$ days.

This indicated 1222 August 15 as the date of the preceding perihelion passage, or 11 months earlier than Hind's date. This is too large a discordance to be possible, so Hind's identification of

the comet of July 1223 with Halley's is erroneous. There was, however, a much more remarkable comet which appeared at the exact epoch indicated by the calculation; and examination shows that the greater part of the statements made concerning it by contemporary writers are quite consistent with its being Halley's, so that the identity is placed beyond reasonable doubt.

The error of the first assumption is so great that it is necessary to recompute the perturbations; this has as yet only been done approximately, the resulting date being 1 day earlier, or August 14; the small difference between this and the preceding result is an illustration of the general proposition that the date of the more remote perihelion passage need only be very roughly known in order to obtain the periodic time correctly.

Pingré's description of this comet is as follows; "En Automne, c'est à-dire aux mois d'Août et de Septembre, on vit une étoile de première grandeur, fort rouge, et accompagnée d'une grande queue qu'elle étendait vers le haut du ciel, en forme d'un cône extrêmement aigu: elle paraissait fort près de la Terre: on l'observa (d'abord) vers le lieu où le soleil se couche au mois de Décembre. Le 15 Août, jour de la première apparition de cette comète (peut-être à Milan) la Lune fut comme morte; elle n'avait plus d'éclat, et elle joignit la comète. On vit ensuite cette comète à l'occident, et même vers le nord, avant la fin du mois d'Août. En Chine on l'observa le 10 Septembre, entre la constellation Kang (les pieds de la Vierge, = ι , κ , λ , θ Virginis), Arcturus et la chevelure de Bérénice: elle disparut le 8 Octobre. Le Père de Mailla dit que les Chinois virent une comète à l'ouest en 1222; à la première lune: c'est sans doute une erreur du copiste, il faut lire à la huitième lune."

The words in parentheses are not part of the original documents, and Pingré's interpolation "d'abord" appears to be erroneous; it is the place where the comet was last seen, not first seen. When the comet extended its tail towards the zenith it must have been nearly vertically above the Sun.

The following is the description in Williams' *Chinese Observations of Comets*:—"In the reign of Ning Tsung, the 15th year of the epoch Kea Ting, the 8th moon, day Kea Woo (= 1222 September 15), a comet appeared in Yew She Te (η , τ , ν Boötis). Its luminous envelope was 30 cubits long. Its body was small, like the planet Jupiter. It was seen for two months. It passed through Te (α , β , γ , ι Libræ), Fang (β , δ , π Scorpii), and Sing (Antares, etc.), and then disappeared." (Williams, in this and other places, erroneously gives ν Boötis instead of ν as one of the components of Yew She Te.)

Most of the above statements are satisfied by the following orbit, in which the longitude of perihelion is that actually derived from the perturbations, and the date of perihelion passage is 8 days later than that indicated above; a perihelion distance somewhat greater than the present value has been used. Hind found that this was also indicated in the return of 1066.

Elements in old System of Elements.

$$\begin{array}{rcl}
 T & = & 1222. \text{ August } 22. \\
 \varpi & = & 296^{\circ}.5 \\
 \Omega & = & 42 \\
 i & = & 16.5 \\
 q & = & 0.67
 \end{array}
 \left. \vphantom{\begin{array}{rcl} T \\ \varpi \\ \Omega \\ i \\ q \end{array}} \right\} \text{Equinox of } 1222.$$

Motion retrograde.

We have had to assume that the date when the comet was first seen in China (in the region η , τ , ν Boötis) should be one lunar month earlier than that given, or August 17 instead of September 15. Our reasons are: (1) it is distinctly stated that the comet was seen for two months in China, the final date being October 8, the position then being in Scorpio; (2) we cannot make this a month later, for in November Scorpio was invisible; (3) further, we are distinctly told that the comet was a splendid object in Europe in mid-August, so that it is most improbable that the Chinese should have missed it till a month later; (4) further, its geocentric motion was at that time so rapid that had it been in Boötes in mid-September it would have been a morning-star on August 15, which it apparently was not.

Another difficulty that we have to surmount is the phrase "La Lune joignit la comète." It was doubtless this phrase that led Pingré to the conclusion that the motion was from south to north, which is in opposition to the Chinese account. Considering the vagueness of all European cometary observations at that epoch, we may look on the phrase as sufficiently satisfied by the fact that the Moon (at her first quarter) and comet were symmetrically placed above the western horizon an hour after sunset, though separated by some 70° . There is one more point to notice. The comet is said to have been near Antares when last seen (October 8). Our chart shows that it was then in Libra, but the head was too near the Sun to be seen; the tail would point to Antares, and, considering its length, it may well have reached that star. It will be seen that we have given the European observations most weight for dates, the Chinese most weight for the track among the stars. This conclusion is in agreement with that reached in the case of other comets, including that of 1301. We have thus shown that the observations are satisfied with no further alteration than they would in any case require to render them self-consistent, and the exact coincidence of date leaves scarcely any room for doubt as to the identity. Pingré was prevented from making this identification by not knowing the wide range through which the period of Halley can alter, and doubtless his inference that the comet moved at first from south to north put Hind off the track.

It is of interest to point out that the revolution 1222-1301 is the longest on record, being 79 years 2 months. It just exceeds that from 1066 to 1145, which was hitherto looked on as the longest. The round now being accomplished is the shortest on

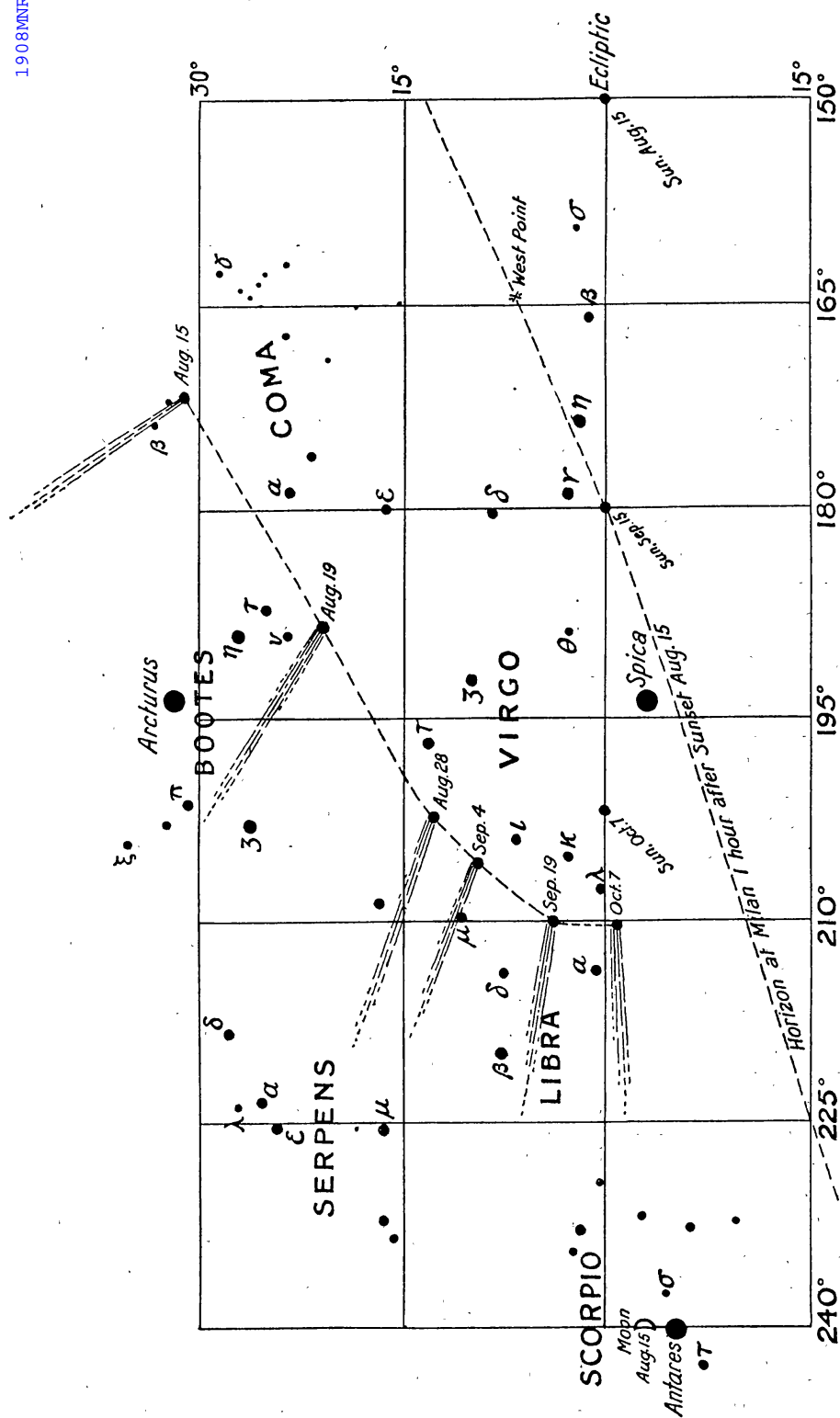


CHART SHOWING THE PATH OF HALLEY'S COMET. 1222.
Longitudes referred to Equinox of 1222.

record, lasting only 74 years $5\frac{1}{2}$ months, a difference of nearly 5 years.

Though the computations for the revolutions 1066–1145, 1145–1222, are still incomplete, enough has been done to make it extremely probable that Hind's identification is correct in each case, though his elements of the apparition of 1066 clearly need modification. They differ more widely from the present elements than perturbations will account for. However, the positions are so vaguely recorded that large alterations in them are possible.

[*Note added January 28.*—Since writing this paper we have done further work on the revolution 1145–1222, the result of which tends to the conclusion that the date of perihelion passage in 1222 was in September rather than August. We are therefore somewhat doubtful whether the date September 15 for the comet being in Yew She Te may not be right after all; if so, the European dates require alteration. We shall return to this point in a subsequent paper. It does not weaken the conclusion as to the identity of the comet of 1222 with Halley's.]

Tables have now been constructed giving the values of the definite integrals for Jupiter and Saturn for the outer half of the orbit. To use them, we have first to find the number of days from perihelion to the extremities of the minor axis, which is done by multiplying the period in days by the number whose log is 8.9825. Hence we find Julian days of passing the points $u = 90^\circ$, $u = 270^\circ$, and then find g' for Jupiter and Saturn by the formulæ—

$$g'_{\frac{1}{2}} = \frac{(\text{Julian Day} - 2390645) 360^\circ}{4332.85},$$

$$g'_{\frac{1}{2}} = \frac{(\text{Julian Day} - 2388249) 360^\circ}{10760.7}.$$

Then the tables give the values of $\int dn$, $\int d\varpi$, $\int d\zeta$, corresponding to the values of g' at first and third quadrant. The algebraical sum of the quantities has to be taken and added to that deduced from the mechanical quadratures.

The numbers from the tables should be modified as follows:—

Multiply $\int dn$ by the number whose log is $\frac{5}{2} [1.2585 - \log a]$, the other columns by the number whose log is $[1.2585 - \log a]$, where a is given its value for the revolution under consideration. This does not apply to the constant part of $\int d\varpi$, $\int d\zeta$, which is independent of a .

Table of the definite integrals $\int dn$, $\int d\varpi$, $\int d\zeta$, for Jupiter and Saturn between the values 90° and 270° of u , the comet's eccentric anomaly.

	Jupiter, $u = 90^\circ$.			Jupiter, $u = 270^\circ$.			Saturn, $u = 90^\circ$.			Saturn, $u = 270^\circ$.		
g'	$\int dn$	$\int d\varpi$	$\int d\zeta$	$\int dn$	$\int d\varpi$	$\int d\zeta$	$\int dn$	$\int d\varpi$	$\int d\zeta$	$\int dn$	$\int d\varpi$	$\int d\zeta$
0	+ "3320	- 306"	+ 5434"	+ "0932	+ 23"	- 125"	+ "0808	+ 38"	+ 1143"	- "0284	+ 77"	- 18"
10	+ "3476	- 253	+ 5955	+ "0986	+ 94	- 121	+ "0743	+ 51	+ 959	- "0382	+ 77	- 12
20	+ "3595	- 191	+ 6237	+ "0959	+ 161	- 113	+ "0670	+ 63	+ 742	- "0476	+ 75	- 7
30	+ "3616	- 124	+ 6306	+ "0852	+ 222	- 104	+ "0588	+ 72	+ 501	- "0567	+ 70	0
40	+ "3559	- 53	+ 6148	+ "0673	+ 275	- 85	+ "0498	+ 78	+ 246	- "0650	+ 62	+ 6
50	+ "3426	+ 19	+ 5775	+ "0426	+ 316	- 67	+ "0406	+ 81	- 14	- "0723	+ 52	+ 12
60	+ "3233	+ 90	+ 5224	+ "0133	+ 347	- 45	+ "0319	+ 82	- 271	- "0783	+ 40	+ 17
70	+ "2976	+ 156	+ 4492	- "0210	+ 367	- 24	+ "0238	+ 80	- 510	- "0828	+ 27	+ 22
80	+ "2681	+ 216	+ 3649	- "0573	+ 374	- 1	+ "0160	+ 75	- 729	- "0856	+ 13	+ 26
90	+ "2351	+ 268	+ 2704	- "0953	+ 368	+ 20	+ "0092	+ 68	- 924	- "0868	0	+ 30
100	+ "2001	+ 310	+ 1700	- "1331	+ 352	+ 41	+ "0033	+ 60	- 1092	- "0867	- 13	+ 32
110	+ "1638	+ 343	+ 656	- "1704	+ 326	+ 59	- "0014	+ 50	- 1224	- "0850	- 25	+ 34
120	+ "1271	+ 367	- 398	- "2057	+ 291	+ 77	- "0048	+ 39	- 1321	- "0820	- 37	+ 34
130	+ "0919	+ 380	- 1406	- "2379	+ 250	+ 98	- "0070	+ 27	- 1382	- "0779	- 48	+ 34
140	+ "0581	+ 382	- 2386	- "2667	+ 201	+ 105	- "0080	+ 14	- 1409	- "0726	- 57	+ 33
150	+ "0267	+ 374	- 3290	- "2913	+ 148	+ 116	- "0079	+ 1	- 1404	- "0666	- 65	+ 31
160	- "0016	+ 356	- 4106	- "3116	+ 91	+ 122	- "0066	- 11	- 1365	- "0600	- 71	+ 28
170	- "0261	+ 329	- 4815	- "3267	+ 32	+ 128	- "0040	- 23	- 1280	- "0529	- 75	+ 25
180	- "0462	+ 294	- 5396	- "3364	- 26	+ 129	- "0003	- 35	- 1179	- "0451	- 78	+ 21
190	- "0619	+ 252	- 5852	- "3409	- 84	+ 126	+ "0044	- 46	- 1042	- "0374	- 78	+ 16
200	- "0724	+ 204	- 6159	- "3400	- 141	+ 121	+ "0100	- 56	- 879	- "0298	- 77	+ 11
210	- "0774	+ 149	- 6308	- "3334	- 195	+ 113	+ "0165	- 64	- 692	- "0224	- 73	+ 5
220	- "0769	+ 89	- 6299	- "3213	- 244	+ 103	+ "0236	- 71	- 485	- "0152	- 68	0
230	- "0710	+ 27	- 6155	- "3046	- 287	+ 88	+ "0312	- 76	- 267	- "0085	- 61	- 5
240	- "0583	- 36	- 5776	- "2817	- 321	+ 70	+ "0394	- 80	- 29	- "0026	- 53	- 11
250	- "0401	- 99	- 5258	- "2545	- 349	+ 51	+ "0478	- 81	+ 207	+ "0023	- 43	- 17
260	- "0164	- 161	- 4582	- "2234	- 366	+ 31	+ "0558	- 80	+ 442	+ "0064	- 31	- 22
270	+ "0120	- 220	- 3765	- "1890	- 375	+ 9	+ "0638	- 76	+ 668	+ "0094	- 19	- 26
280	+ "0453	- 273	- 2817	- "1517	- 371	- 13	+ "0710	- 70	+ 877	+ "0110	- 5	- 29
290	+ "0819	- 319	- 1766	- "1131	- 356	- 36	+ "0773	- 61	+ 1052	+ "0110	+ 9	- 32
300	+ "1210	- 355	- 646	- "0740	- 329	- 57	+ "0827	- 51	+ 1211	+ "0097	+ 23	- 33
310	+ "1617	- 380	+ 523	- "0359	- 291	- 78	+ "0865	- 39	+ 1322	+ "0067	+ 37	- 33
320	+ "2023	- 392	+ 1691	+ "0003	- 242	- 94	+ "0891	- 25	+ 1391	+ "0023	+ 49	- 33
330	+ "2411	- 390	+ 2740	+ "0323	- 184	- 109	+ "0897	- 10	+ 1406	- "0036	+ 59	- 31
340	+ "2766	- 376	+ 3833	+ "0594	- 118	- 119	+ "0885	+ 6	+ 1370	- "0109	+ 68	- 28
350	+ "3074	- 347	+ 4722	+ "0800	- 49	- 124	+ "0855	+ 23	+ 1260	- "0196	+ 74	- 24

Constant part of $\int d\varpi$ (to be added to tabular quantities) Jupiter - 112",
Saturn - 24", Total - 136".

Constant part of $\int d\zeta$ (to be added to tabular quantities) Jupiter + 2700",
Saturn + 765", Total + 3465".

As an example of the use of these tables we may deduce the values already obtained for the revolution 1835-1910 (*M.N.*, vol. lxvii. pp. 410, 411, 519, 520). For Jupiter the values of g' at $u=90^\circ$, $u=270^\circ$, are $296^\circ.44$, $322^\circ.24$, for Saturn $199^\circ.52$, $214^\circ.74$.

Hence we obtain—

	$\int dn$	$\int d\varpi$	$\int d\zeta$
$2\ 90^\circ$	+ "1068	- 343	- 1056
„ 270°	+ "0074	- 229	- 97
$h\ 90^\circ$	+ "0097	- 55	- 887
„ 270°	- "0190	- 71	+ 3
Sum $2\ 90^\circ$	+ "1142	- 572	- 1153
„ $h\ 90^\circ$	- "0093	- 126	- 884

Now $\log a$ for this revolution = 1.2480, whence we have to multiply $\int dn$ by number whose log is .0262, $\int d\varpi$, $\int d\zeta$ by number whose log is .0105.

Performing the multiplication and adding the constants we obtain—

	$\int dn$	$\int d\varpi$	$\int d\zeta$
$2\ 90^\circ$	+ "1213	- 698	+ 1519
$h\ 90^\circ$	- "0099	- 153	- 141
Results previously obtained $\left\{ \begin{array}{l} 2\ 90^\circ \\ h\ 90^\circ \end{array} \right.$	$\left\{ \begin{array}{l} + "1219 \\ - "0101 \end{array} \right.$	$\left\{ \begin{array}{l} - 698 \\ - 153 \end{array} \right.$	$\left\{ \begin{array}{l} + 1532 \\ - 142 \end{array} \right.$

A more complete investigation of the average motion of the comet's perihelion has been made, using the results of all the revolutions for which the perturbations have now been computed. It appears that the perihelion has a direct motion in longitude at nearly the same rate as the node, so that the periods of the various planets for co-ordinates $x' y'$ may be taken the same as those given for z' , p. 112. The value of ϖ for 1910 should be $305^\circ.64$, not $303^\circ.64$ as printed on p. 112.